

The Shadow and the Object

Dimensional Projection and the Limits of Observation

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ABSTRACT This paper defends a thesis I call *projection realism*: what we observe is, in a precise sense, the projection of a state that has more independent components than the three dimensions of space and one of time in which we observe it, and a projection necessarily discards information. I argue that several episodes usually presented as paradoxes of mathematics and physics, among them the “imaginary” roots of a quadratic, the invisibility of velocity in an instantaneous image, the relativity of simultaneity and the correlations of entangled particles, share one structure: a feature of the projection is mistaken for a feature of the object. I state the thesis formally, distinguish three kinds of projection artefact, and show that the history of unification in physics, from Maxwell and Minkowski to Kaluza and Klein, is a history of enlarging the space of components rather than of refining observation within a fixed one. I then answer four objections: from constructive empiricism, from underdetermination, from Kant’s unknowable thing in itself, and from Bell’s theorem. The conclusion is that Plato’s allegory of the cave, read geometrically, is a better guide to the relation between observation and reality than the Aristotelian conviction that careful observation is sufficient for knowledge, although observation remains necessary.

KEYWORDS Plato’s cave; phenomenon and noumenon; structural realism; projection; imaginary numbers; entanglement; Kaluza–Klein theory; unification

I. INTRODUCTION

Physics and mathematics have repeatedly met objects that look impossible from where we stand and ordinary from somewhere else. A number whose square is negative, a particle that seems to act instantaneously on another kilometres away, a clock that runs slow merely because it moves: each was first received as a scandal and later absorbed into theory. The absorption usually took the same form. A larger space was introduced, the scandalous object was placed in it, and the scandal was found to be an effect of looking at that space from a smaller one.

The purpose of this paper is to make that pattern explicit and to ask what it tells us about reality. My thesis is that the pattern is not an accident of the history of science but a consequence of the relation between observation and its objects. Observation is a projection. It takes a state with many components and returns a few of them, and the components it does not return are not thereby unreal. When we take the projection for the object, the world looks paradoxical, and in that sense, and only in that sense, illusory.

The thesis has an ancient ancestor. In the seventh book of the *Republic*, Plato describes prisoners who see only shadows on the wall of a cave and take them for the whole of what exists (Plato, *Republic* VII). I shall argue that the allegory can be read almost literally as a claim about projections, and that so read it is correct in a way that the opposite conviction, associated with Aristotle, that knowledge is completed by the careful accumulation of observations, is not. Aristotle was

right that knowledge begins with the senses; he was wrong, on this reading, if he thought it could end there.

Section 2 reads the cave geometrically. Section 3 presents four case studies. Section 4 states the thesis. Section 5 classifies the artefacts of projection. Section 6 turns to unification in physics. Section 7 considers objections, and Section 8 returns to Plato and Aristotle.

2. THE CAVE, READ GEOMETRICALLY

Plato's prisoners are chained so that they can see only the wall in front of them. Behind them burns a fire, and between the fire and the prisoners people carry artefacts whose shadows fall on the wall (*Republic* 514a–515c). The prisoners name the shadows, record their regularities, and honour whoever predicts them best. One prisoner is released, turned towards the fire, and finally led into the sunlight, where after a painful period of adjustment he sees things as they are. When he returns, he is worse than the others at the game of shadows, and they conclude that the ascent has damaged him (516e–517a).

Three features of the story matter here. First, the shadows are not false. They are caused by the artefacts and vary lawfully with them; a sufficiently diligent prisoner could build an accurate predictive science of shadows. Second, the shadows are poorer than the artefacts. A shadow is two-dimensional and has lost depth, colour and texture; distinct artefacts may cast identical shadows, and a single artefact, turned, may cast shadows that look like distinct things. Third, the release is a change of *perspective*, not an improvement of eyesight. The prisoner does not see the wall better; he turns around.

In modern terms the shadow is the image of the artefact under a projection: a map from a space of higher dimension to one of lower dimension that keeps some components and discards the rest. The three features just listed are then theorems rather than metaphors. The image is determined by the object (the shadow is not false). The map is not injective, so different objects can share an image (the shadow is poorer). And no amount of refinement within the image recovers what the map discarded; that requires a second map, a different projection, or a model of the space the image came from (release is a change of perspective).

3. FOUR CASES

3.1 *Imaginary numbers*

The equation $x^2 + 4 = 0$ has no real solution: on the real line, no number has a negative square. The Renaissance algebraists nevertheless calculated with square roots of negative numbers. Cardano, who met them in 1545 while dividing ten into two parts with product forty, called the result “as subtle as it is useless” (Cardano 1545); Descartes in 1637 named such quantities *imaginary* (Descartes 1637), and the name has coloured their reception ever since.

The resolution came when Wessel, Argand and Gauss represented complex numbers as points of a plane (Wessel 1799; Argand 1806; Gauss 1831). The number i is then a unit step along a second axis, perpendicular to the real line, and multiplication by i is a rotation through a right angle, so that $i^2 = -1$ because two quarter turns face backwards. Nothing in this picture is imaginary. The solution of $x^2 + 4 = 0$ lies two units along the perpendicular axis; seen from the real line

alone, it is not there at all. Gauss drew the moral himself: had $+1$, -1 and $\sqrt{-1}$ been called the direct, inverse and *lateral* units, the subject would never have seemed obscure (Gauss 1831). The appearance of impossibility was an artefact of searching for the answer in a space with too few dimensions.

3.2 Velocity

A photograph of a moving robot arm and a photograph of the same arm at rest, in the same configuration, are identical. Velocity does not appear in an instantaneous image. Yet the arm's velocity is as real as its position: it determines where the arm will be. Classical mechanics has long recognised this by treating position and velocity (or momentum) as independent coordinates of a state in *phase space*. In the language of this paper, the instantaneous image is a projection of the state that discards the velocity components. We perceive velocity only indirectly, through the way successive projections differ. The robot's speed is, in this modest but exact sense, a dimension we experience without seeing.

3.3 Space and time

Einstein opened his 1905 paper on the electrodynamics of moving bodies with an asymmetry: a magnet moved near a conductor and a conductor moved near a magnet produce the same current, yet the theory of the day described the two cases differently, with an electric field in one and a magnetic force in the other (Einstein 1905). Three years later Minkowski recast the theory as the geometry of a four-dimensional spacetime and declared that "space by itself, and time by itself, are doomed to fade away into mere shadows" (Minkowski 1909). In spacetime, the electric and magnetic fields are components of a single object, and the contraction of moving rods and the dilation of moving clocks are the shortening of projections of a four-dimensional interval that does not change. The word *shadows* was Minkowski's, and it was chosen with care.

3.4 Entanglement

In 1935 Einstein, Podolsky and Rosen considered two particles that have interacted and then separated, and argued that if quantum mechanics is correct, a measurement on one allows the certain prediction of a result on the other without disturbing it; either the second particle possessed the value all along, and the theory is incomplete, or the measurement acted instantaneously at a distance (Einstein, Podolsky and Rosen 1935). Bohr replied that the question was ill posed (Bohr 1935). The exchange continued privately for two decades; in 1954 Pauli, writing to Born, compared Einstein's question to the ancient question "of how many angels are able to sit on the point of a needle" (Born 1971).

Bell showed in 1964 that the question was not idle (Bell 1964). If each particle carries its own hidden variables and answers its own measurement from them, the correlations obey inequalities that quantum mechanics violates; experiments since Aspect's have confirmed the violation, most stringently in 2015 (Aspect, Dalibard and Roger 1982; Hensen et al. 2015). The lesson usually drawn is that no theory of locally carried hidden variables can be correct. The lesson I wish to draw is complementary: the description of an entangled pair as *two* objects, each with its own state, is itself a projection. The quantum state of two particles is not a pair of states in three-dimensional space but a single state over a space of higher dimension, the configuration space of both together, and it cannot in general be written as a product of two single-particle states (Schrödinger 1935).

Seen in space, the pair appears as two things that change together at a distance; seen in its own space, it is one thing that changes.

4. THE PROJECTION THESIS

The four cases share a structure, which I now state.

Definition 4.1. Let S be a set of possible states of a system, and O a set of possible observations. An *observation map* is a function $\pi: S \rightarrow O$ assigning to each state what an observer of a given kind records of it. The map is a *projection* if it is surjective and not injective: every observation is of some state, and some distinct states are observed identically.

When S and O are vector spaces and π is linear, the failure of injectivity has a precise measure: the kernel of π , the set of state differences that make no observable difference, has dimension $\dim S - \dim O$. Every direction in the kernel is a way in which reality can vary while appearance stays fixed.

PROJECTION REALISM. (i) The states of physical systems are elements of spaces with more independent components than the four of space and time. (ii) Observation, as performed by creatures like us and by instruments built on the same model, is a projection of these states. (iii) Hence the observed world systematically underdetermines the world observed, and features of the projection are liable to be mistaken for features of the object.

Clause (i) is a claim about ontology; clause (ii) is a claim about epistemology; clause (iii) follows from them by the definition of a projection. Kant's distinction between the *phenomenon*, the object as it appears, and the *noumenon*, the object as it is in itself (Kant 1781/1787), maps onto the distinction between the image $\pi(s)$ and the state s . But projection realism departs from Kant in one important respect, to which I return in Section 7: the components discarded by π are not in principle unknowable. They are known in the way the depth of the artefacts is known to the released prisoner, by changing perspective and reconstructing.

Projection realism has a close relative in contemporary metaphysics of quantum mechanics. *Wave-function realism* holds that the fundamental arena of a quantum world of N particles is a space of $3N$ dimensions, and that the three-dimensional world of experience is derivative (Albert 1996; Ney 2021). The present thesis is more general and more modest: more general, because it applies equally to classical kinematics and to the algebra of numbers; more modest, because it does not say which higher-dimensional space is fundamental, only that ours is a projection of one.

5. THREE ARTEFACTS OF PROJECTION

If the thesis is correct, certain kinds of appearance should arise wherever a state is observed through a projection. Three are worth distinguishing, because each has been mistaken, at some point, for a fact about the world.

Creation and annihilation. Consider a rod of unit length in a space with one direction hidden from the observer, and let it rotate steadily in a plane that contains that direction. Its length never changes. Its shadow, however, has length $|\cos \theta|$: it shrinks, vanishes when the rod is perpendicular to the visible space, and reappears pointing the other way. An observer confined to the shadow would describe the destruction of a rod and the creation of its reversed twin. Abbott dramatised exactly this in 1884: a sphere passing through a two-dimensional world appears to its inhabitants as a point that becomes a circle, grows, shrinks and vanishes (Abbott 1884).

Inversion. Continue the same rotation to a half turn and apply it to a whole three-dimensional object. Seen through the projection, the object has become its mirror image, a left hand has become a right hand, and no motion within three dimensions could have achieved this. Kant made incongruent counterparts, such as the left and the right hand, an argument about the nature of space (Kant 1768); Möbius observed in 1827 that a rotation through a fourth dimension would carry one into the other (Möbius 1827).

Action at a distance. Let a single state have two groups of components that are observed in two different places, and let a third component couple them. A change in the coupling component alters both groups in the same step. Observed through two projections, each of which sees one group only, this appears as an instantaneous correlation between two separated things. This is the structure of Section 3.4.

A fourth familiar artefact deserves a remark, although it is a matter of reference frame rather than of dimension. The Coriolis force, which deflects winds and projectiles on the rotating Earth (Coriolis 1835), is real in the sense that it must be included in any correct calculation made in the rotating frame; yet seen from an inertial frame no such force acts. Here too a feature of a viewpoint is experienced as a feature of the world. The lesson generalises: what is observed depends on how it is observed, and the dependence can masquerade as a force, a creation, an inversion or an influence.

6. UNIFICATION AS A PROBLEM OF PERSPECTIVE

Physics has a standing ambition to unify: to show that phenomena treated by separate theories are aspects of one. It is instructive to ask how its past unifications were achieved.

Date	What was unified	By what enlargement
1865	electricity, magnetism, light	one field with more components (Maxwell 1865)
1905–1908	space and time; electric and magnetic fields	a fourth dimension (Einstein 1905; Minkowski 1909)
1921–1926	gravitation and electromagnetism (classically)	a fifth dimension (Kaluza 1921; Klein 1926)
1961–1968	electromagnetic and weak interactions	internal components of every field (Glashow 1961; Weinberg 1967)
1995	the known string theories	an eleventh dimension (Witten 1995)

Kaluza's result is the clearest instance. Writing Einstein's field equations in five dimensions instead of four, he found that the additional components of the metric reproduce Maxwell's equations of electromagnetism, together with one additional scalar field (Kaluza 1921). Klein then

showed that if the fifth dimension is a small circle, electric charge corresponds to momentum along it (Klein 1926). A quantity that appears fundamental and irreducible in four dimensions is, in five, the projection of a motion.

The pattern invites a hypothesis, which I state as a hypothesis and not as a result: *the remaining failures of unification, above all the failure to reconcile gravitation with quantum theory, may be failures of perspective rather than of physics*. We may be trying to fit into four dimensions a state that, like the roots of $x^2 + 4 = 0$, the velocity of the robot and the entangled pair, does not fit there. The hypothesis has consequences. It recommends that we look for unification by enlarging the space of components and searching for the projection that yields the observed theories, rather than by further refinement within the observed space. It is, of course, a heuristic, and heuristics can fail; the proliferation of candidate higher-dimensional theories shows that enlarging the space is not sufficient. But it has been the right heuristic at every decisive step so far.

7. OBJECTIONS AND REPLIES

7.1 *The empiricist objection*

A constructive empiricist will say that science aims only at empirical adequacy, and that belief in unobservable components is an optional extra (van Fraassen 1980). On this view the discarded components are idle: if they make no observable difference, there is nothing to be right or wrong about.

The reply is that the discarded components are not idle; they are what make the projections cohere. The velocity of the robot is not visible in any photograph, but it explains the relation between successive photographs. The hidden components of Dirac's equation were not observed in 1928, but they entailed the positron, observed in 1932 (Dirac 1928; Anderson 1933). A component is discarded by one projection, not by all; its postulation is tested by the others. The empiricist is right that we have no access to reality except through projections. He is wrong to conclude that the projections are all there is to be right about.

7.2 *The objection from underdetermination*

Given a set of observations, infinitely many higher-dimensional states project onto them. How, then, can we claim knowledge of any particular one? The objection is correct as far as it goes, and the thesis concedes it: a single projection underdetermines the state. But underdetermination is reduced by multiplying independent projections, as a second light source fixes the height of a ball whose shadow alone does not, and by demanding that the lifted theory be simpler than the projected one: linear where the observations were not, first order where they were second order. The lifts that history has vindicated had exactly this property.

7.3 *The Kantian objection*

Kant held that the thing in itself is unknowable: we know objects only as they appear under the forms of our sensibility (Kant 1781/1787). Is projection realism not simply Kant, restated?

It is not, for the thesis claims that the noumenal components are knowable *in their structure*. The released prisoner does not perceive the artefacts with some new faculty; he reconstructs them from their relations. Structural realists have argued that what successful theories preserve

through revolutions is precisely such structure (Worrall 1989; Ladyman 1998). Projection realism is a structural realism about the space of components: we know the dimension of the hidden space, the maps between its projections and the laws that hold in it, even when we do not perceive its elements.

7.4 *The objection from Bell's theorem*

Finally, it may be objected that to describe the entangled pair as one object in a higher-dimensional space is to reintroduce hidden variables, which Bell's theorem has refuted. The objection conflates two theses. Bell's theorem refutes hidden variables that are carried separately by each particle and act locally (Bell 1964). It says nothing against a description in which the relevant variables belong to the pair as a whole; indeed Bell himself regarded Bohm's 1952 theory, in which they do, as a serious possibility (Bohm 1952; Bell 1987). The price of such a description is well known: in three-dimensional terms it is non-local, and reconciling it with relativity requires care. Projection realism does not remove this price. It reinterprets it: what is non-local in the projection may be local, or simply unitary, in the space from which the projection is taken.

8. PLATO AND ARISTOTLE

It is customary to contrast Plato, for whom the sensible world is an image of the intelligible, with Aristotle, for whom knowledge of universals arises by induction from perception (Aristotle, *Posterior Analytics* II.19). Modern science is often said to have followed Aristotle, and in its method it has: it begins with observation and returns to it. But the cases of this paper suggest that on the question of what observation *reveals*, Plato saw further. The complex plane, the phase space of mechanics, Minkowski's spacetime and the configuration space of quantum mechanics are none of them observed. Each was introduced to make sense of observations that, taken at face value, were paradoxical.

The correct position is therefore neither Aristotle's nor, in every detail, Plato's. Observation is necessary: without the shadow, the prisoner would have nothing to reconstruct. It is not sufficient: the shadow is the image of an object richer than itself, and to know the object is to reconstruct what the image discards. The instrument of reconstruction is mathematics, which, on this view, is the craft of placing a thing in a space of orthogonal components and finding the components in which its behaviour is simplest.

9. CONCLUSION

I have argued that observation is a projection, that projections discard information, and that the paradoxes which have punctuated the history of mathematics and physics are, in a characteristic family of cases, the result of mistaking the projection for the object. Imaginary numbers are real rotations seen from a line; velocity is a component invisible to an instantaneous image; the contraction of moving rods is the shortening of a projection; and two entangled particles may be one object seen twice. The same pattern governs the history of unification, which has advanced by enlarging the space of components.

If this is right, reality is not three-dimensional, nor four-dimensional; it is a state with more

components than we can perceive, and the world of experience is its projection. That world is not false, but it is incomplete, and it becomes an illusion precisely when we take it for the whole. Plato's prisoners were not deceived by the shadows. They were deceived by the belief that there was nothing behind them.

REFERENCES

- Abbott, E. A. (1884). *Flatland: A Romance of Many Dimensions*. London: Seeley.
- Albert, D. Z. (1996). Elementary quantum metaphysics. In J. T. Cushing, A. Fine and S. Goldstein (eds), *Bohmian Mechanics and Quantum Theory: An Appraisal*, 277–284. Dordrecht: Kluwer.
- Anderson, C. D. (1933). The positive electron. *Physical Review* 43, 491–494.
- Argand, J.-R. (1806). *Essai sur une manière de représenter les quantités imaginaires dans les constructions géométriques*. Paris.
- Aristotle. *Posterior Analytics*, II.19. Trans. J. Barnes (2nd edn). Oxford: Clarendon Press, 1994.
- Aspect, A., Dalibard, J. and Roger, G. (1982). Experimental test of Bell's inequalities using time-varying analyzers. *Physical Review Letters* 49, 1804–1807.
- Bell, J. S. (1964). On the Einstein Podolsky Rosen paradox. *Physics* 1, 195–200.
- Bell, J. S. (1987). *Speakable and Unspeakable in Quantum Mechanics*. Cambridge: Cambridge University Press.
- Bohm, D. (1952). A suggested interpretation of the quantum theory in terms of “hidden” variables, I and II. *Physical Review* 85, 166–193.
- Bohr, N. (1935). Can quantum-mechanical description of physical reality be considered complete? *Physical Review* 48, 696–702.
- Born, M. (ed.) (1971). *The Born–Einstein Letters*. London: Macmillan.
- Cardano, G. (1545). *Ars Magna*. Nuremberg.
- Coriolis, G.-G. de (1835). Sur les équations du mouvement relatif des systèmes de corps. *Journal de l'École polytechnique* 15, 142–154.
- Descartes, R. (1637). *La Géométrie*. Leiden.
- Dirac, P. A. M. (1928). The quantum theory of the electron. *Proceedings of the Royal Society A* 117, 610–624.
- Einstein, A. (1905). Zur Elektrodynamik bewegter Körper. *Annalen der Physik* 17, 891–921.
- Einstein, A., Podolsky, B. and Rosen, N. (1935). Can quantum-mechanical description of physical reality be considered complete? *Physical Review* 47, 777–780.
- Gauss, C. F. (1831). Anzeige: Theoria residuorum biquadraticorum, commentatio secunda. *Göttingische gelehrte Anzeigen*, 625–638.
- Glashow, S. L. (1961). Partial-symmetries of weak interactions. *Nuclear Physics* 22, 579–588.
- Hensen, B. et al. (2015). Loophole-free Bell inequality violation using electron spins separated by 1.3 kilometres. *Nature* 526, 682–686.
- Kaluza, T. (1921). Zum Unitätsproblem der Physik. *Sitzungsberichte der Preussischen Akademie der Wissenschaften*, 966–972.
- Kant, I. (1768). Von dem ersten Grunde des Unterschiedes der Gegenden im Raume. Königsberg.
- Kant, I. (1781/1787). *Critique of Pure Reason*. Trans. P. Guyer and A. W. Wood. Cambridge: Cambridge University Press, 1998.
- Klein, O. (1926). Quantentheorie und fünfdimensionale Relativitätstheorie. *Zeitschrift für Physik* 37, 895–906.

- Ladyman, J. (1998). What is structural realism? *Studies in History and Philosophy of Science* 29, 409–424.
- Maxwell, J. C. (1865). A dynamical theory of the electromagnetic field. *Philosophical Transactions of the Royal Society* 155, 459–512.
- Minkowski, H. (1909). Raum und Zeit. *Physikalische Zeitschrift* 10, 104–111.
- Möbius, A. F. (1827). *Der barycentrische Calcul*. Leipzig: Barth.
- Ney, A. (2021). *The World in the Wave Function: A Metaphysics for Quantum Physics*. New York: Oxford University Press.
- Plato. *Republic*. Trans. G. M. A. Grube, rev. C. D. C. Reeve. Indianapolis: Hackett, 1992.
- Schrödinger, E. (1935). Discussion of probability relations between separated systems. *Mathematical Proceedings of the Cambridge Philosophical Society* 31, 555–563.
- van Fraassen, B. C. (1980). *The Scientific Image*. Oxford: Clarendon Press.
- Weinberg, S. (1967). A model of leptons. *Physical Review Letters* 19, 1264–1266.
- Wessel, C. (1799). Om directionens analytiske betegning. *Nye Samling af det Kongelige Danske Videnskabelnes Selskabs Skrifter* 5, 469–518.
- Witten, E. (1995). String theory dynamics in various dimensions. *Nuclear Physics B* 443, 85–126.
- Worrall, J. (1989). Structural realism: the best of both worlds? *Dialectica* 43, 99–124.