

Hidden Components

Projection, Extra Dimensions and Entanglement in Physical Description

ROY GONZALEZ

Independent researcher

ABSTRACT Physical states generally carry more independent components than an observation in three-dimensional space at one instant can display. We develop this observation systematically as a working principle: that the observed quantities of a system are a projection of a state in a larger space of components, and that apparent anomalies in the observed quantities often signal components that the projection has discarded. We illustrate the principle at four levels. In kinematics, the velocity of a manipulator is invisible in any single configuration and appears, in a five-dimensional homogeneous description, as a shear generated by the time axis; in special relativity the same velocity is a hyperbolic rotation. In the Dirac theory, the requirement of a first-order equation forces spinor components that were not observed in 1928 and whose consequences, spin and the positron, were. In Kaluza–Klein theory, a fifth dimension turns electric charge into momentum and a single field into a tower of masses, now constrained by collider and torsion-balance experiments. For entangled pairs, the Bell–CHSH inequality excludes descriptions by two separately carried sets of variables; we give an explicit real twelve-component model, one state observed through two projections, in which the correlations arise from a single coupling acting in one step, and we state its costs. We close with the observation that the historical unifications of physics were achieved by enlarging the space of components, and discuss what would be required to turn this pattern into a testable programme.

KEYWORDS homogeneous coordinates; Galilean and Lorentz boosts; Dirac equation; Kaluza–Klein theory; extra dimensions; entanglement; Bell inequalities; non-separability; unification

I. INTRODUCTION

An instantaneous image of a physical system records positions. It does not record velocities, spins, phases, internal quantum numbers or the correlations between separated parts; all of these must be inferred from sequences of images or from the statistics of many. In this limited but exact sense every observation is a projection of the state onto a subspace of what the state contains.

This paper takes that fact as a working principle and follows it through several levels of physical theory. The principle is not new in any of its instances. Phase space treats velocity as an independent coordinate (Gibbs 1902); Minkowski’s spacetime unites space and time (Minkowski 1909); Dirac’s equation requires more components than the Schrödinger wave function (Dirac 1928); Kaluza and Klein added a fifth dimension (Kaluza 1921; Klein 1926). What we wish to emphasise is the common structure, and in particular a heuristic that follows from it: when observed quantities behave anomalously, being non-linear where one expects linearity, second order where one expects first order, or correlated at spacelike separation, it is worth asking which components the observation has discarded.

We use natural units ($\hbar = c = 1$) except where stated. All numerical values were computed in double precision.

2. KINEMATICS: VELOCITY AS A HIDDEN COMPONENT

2.1 Homogeneous coordinates

Rigid motions of $\mathbb{R}^3$ are not linear, since translations are not. In homogeneous coordinates (x, w) , $w = 1$, a translation by d becomes the shear $x \mapsto x + dw$, and every rigid motion is a 4×4 matrix $\begin{pmatrix} R & d \\ 0 & 1 \end{pmatrix}$. Serial manipulators are described as products of such matrices, one per joint (Denavit and Hartenberg 1955). The fourth coordinate is never observed, since $w = 1$ identically, but every translation of the arm is written through it.

2.2 A fifth coordinate for velocity

Add time as a coordinate and write an event as (x, w, t) . A body moving with velocity v is transported by the matrix

$$D_v = \begin{pmatrix} I & 0 & v \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = \exp \left(\sum_i v_i E_{i,t} \right), \quad x \mapsto x + vt, \quad (2.1)$$

a shear generated by the time axis. The matrices $\begin{pmatrix} R & a & v \\ 0 & 1 & 0 \\ 0 & \tau & 1 \end{pmatrix}$ form a faithful representation of the Galilean group, with composition law $v = v_1 + R_1 v_2$. In this description the w -column carries where a body is and the t -column carries how it moves.

2.3 An example

Consider a three-joint arm (base yaw q_0 , shoulder pitch q_1 , elbow pitch q_2) with links $l_1 = 0.5$ m and $l_2 = 0.4$ m, at $(q_0, q_1, q_2) = (30^\circ, 20^\circ, 50^\circ)$. Its hand is at

$$(x, y, z) = (0.52538, 0.30333, 0.54689) \text{ m.}$$

Three motions from this configuration give the hand velocities in Table 1.

Joint rates, deg/s	Hand velocity, m/s	Speed, m/s
(0, 10, -10)	(-0.02585, -0.01492, 0.08200)	0.08727
(0, 90, -90)	(-0.23263, -0.13431, 0.73803)	0.78540
(60, 0, 0)	(-0.31764, 0.55017, 0)	0.63529

Table 1: Hand velocities of a three-joint arm from one configuration. All three motions give the same instantaneous image.

All three motions share the same instantaneous image. Transported by (2.1) for 0.2 s, the images of the first two separate: (0.52021, 0.30034, 0.56329) and (0.47885, 0.27646, 0.69449). The true hand positions after 0.2 s of joint motion are (0.51996, 0.30020, 0.56318) and (0.45970, 0.26541, 0.68371); the drift is the tangent to the true path. The configuration itself is also underdetermined by the image: the same hand is reached with the shoulder at 64.06809° and the elbow at -50° . For an industrial arm with six revolute joints the state has six angles and six rates, twelve components, of which an image of the hand records three.

2.4 The relativistic form

In special relativity the same velocity is a rotation. With $K = E_{x,t} + E_{t,x}$ one has $K^3 = K$, and

$$\Lambda(\varphi) = e^{\varphi K} = I + \sinh \varphi K + (\cosh \varphi - 1)K^2, \quad v = \tanh \varphi, \quad (2.2)$$

the hyperbolic counterpart of Rodrigues' rotation formula. As $c \rightarrow \infty$ the boost contracts to the shear (2.1) (İnönü and Wigner 1953). For the faster motion of Table 1, $v = 0.78540$ m/s, the rapidity is $\varphi = 2.6 \times 10^{-9}$: the worldline of the hand is tilted towards space by that angle. Einstein motivated the theory by an asymmetry, the magnet and conductor that produce one current but were described by two mechanisms (Einstein 1905); Minkowski's geometry removed it by making the electric and magnetic fields components of one tensor, and called space and time by themselves "mere shadows" (Minkowski 1909).

3. THE HIDDEN COMPONENTS OF THE ELECTRON

3.1 A first-order equation needs more components

The relativistic energy relation $E^2 = p^2 + m^2$ gives the Klein–Gordon equation $(\square + m^2)\psi = 0$, second order in time and with one component. Dirac sought a first-order operator whose square is this one (Dirac 1928). Writing $P = i\gamma^\mu \partial_\mu$, the requirement $(P + m)(P - m) = -(\square + m^2)$ holds if and only if the coefficients satisfy

$$\gamma^\mu \gamma^\nu + \gamma^\nu \gamma^\mu = 2\eta^{\mu\nu} I, \quad (3.1)$$

and in $3 + 1$ dimensions the smallest matrices with this property are 4×4 . More generally, in d dimensions the Clifford relations are realised by matrices of size $2^{\lfloor d/2 \rfloor}$, built recursively by $\gamma_a^{(n+2)} = X \otimes \gamma_a^{(n)}, \gamma_{n+1}^{(n+2)} = Y \otimes I, \gamma_{n+2}^{(n+2)} = Z \otimes I$ from the Pauli matrices (Brauer and Weyl 1935). The price of reducing the order of the equation is the introduction of new components.

3.2 What the components turned out to be

The four components of the Dirac spinor carry spin, with gyromagnetic ratio $g = 2$, and a second solution branch, which Dirac eventually interpreted as an antiparticle; the positron was observed by Anderson in 1932 (Anderson 1933). The components are not uniformly visible. For the hydrogen ground state, with $\gamma = \sqrt{1 - \alpha^2} = 0.9999733$, the "large" components carry a fraction $(1 + \gamma)/2 = 99.9987\%$ of the probability and the "small" components $(1 - \gamma)/2 = 0.0013\%$. A non-relativistic measurement sees essentially one two-component spinor; the remaining components are there, and they account for the fine structure of the spectrum.

4. A FIFTH DIMENSION

4.1 Kaluza and Klein

In 1921 Kaluza wrote general relativity in five dimensions and found that, under a cylinder condition, the extra components of the metric obey Maxwell's equations, with an additional scalar field (Kaluza 1921). In 1926 Klein proposed that the fifth dimension is a circle of small radius

R and related electric charge to momentum along it (Klein 1926). The same mechanism acts on matter fields. If ψ satisfies the five-dimensional Klein–Gordon equation and $\psi = e^{inx_5/R} g(x^\mu)$, then

$$\left(\square_4 + m^2 + \frac{n^2}{R^2}\right)g = 0, \quad m_n = \sqrt{m^2 + n^2/R^2}, \quad (4.1)$$

so a single five-dimensional field appears in four dimensions as a tower of particles whose masses record their momentum in the hidden direction. By the result of Section 3, every solution of the five-dimensional Dirac equation also solves (4.1).

4.2 Experimental status

No Kaluza–Klein partner of any Standard Model particle has been observed. In models with universal extra dimensions (Appelquist, Cheng and Dobrescu 2001), searches at the Large Hadron Collider at $\sqrt{s} = 8$ TeV exclude compactification scales $1/R$ below about 950 GeV for large cut-offs (ATLAS Collaboration 2015), that is $R \lesssim 2 \times 10^{-19}$ m. Dimensions accessible to gravity alone are constrained by torsion-balance tests of the inverse-square law, which find no deviation down to 52 μm (Lee et al. 2020). Extra dimensions, if they exist, are either very small or not felt by the fields we observe directly.

5. ENTANGLEMENT

5.1 The question

Einstein, Podolsky and Rosen argued that quantum mechanics, if it predicts with certainty the result of a measurement on a distant particle without disturbing it, must be incomplete (Einstein, Podolsky and Rosen 1935); Bohr disputed the premise (Bohr 1935). Schrödinger introduced the term entanglement for the states in question and observed that they cannot be written as products of states of the parts (Schrödinger 1935). Einstein’s later phrase for the dilemma, “spooky action at a distance”, is from a letter to Born of 1947, and the Copenhagen reply is typified by Pauli’s remark to Born in 1954 that such questions resemble the problem of how many angels fit on the point of a needle (Born 1971).

5.2 Bell’s theorem and its tests

Let each of two separated stations measure a dichotomic observable along an angle, a at A and b at B , and let $E(a, b)$ be the mean product of the outcomes. If each outcome is a function of the local setting and of a variable λ shared at the source, then

$$|S| = |E(a, b) - E(a, b') + E(a', b) + E(a', b')| \leq 2 \quad (5.1)$$

(Bell 1964; Clauser, Horne, Shimony and Holt 1969). For the spin singlet quantum mechanics predicts $E(a, b) = -\cos(a - b)$ and $|S| = 2\sqrt{2}$ at $a = 0$, $a' = 90^\circ$, $b = 45^\circ$, $b' = 135^\circ$, the maximum allowed by the theory (Tsirelson 1980). A model in which each particle carries its own copy of λ , with $A = \text{sgn} \cos(a - \lambda)$ and $B = -\text{sgn} \cos(b - \lambda)$, gives the correlations of Table 2 and saturates the bound at $|S| = 2$. Experiments have established the violation of (5.1) since Aspect, Dalibard and Roger 1982, and without the principal loopholes since 2015 (Hensen et al. 2015; Giustina et al. 2015; Shalm et al. 2015).

$a - b$	0°	45°	90°	135°	180°
two separately carried variables	-1	-0.5	0	0.5	1
quantum mechanics and experiment	-1	-0.7071	0	0.7071	1

Table 2: Correlation $E(a, b)$ as a function of the angle between the settings.

What the theorem excludes is precisely the description by two objects, each with its own variables. It does not exclude variables that belong to the pair as a whole; the de Broglie–Bohm theory is an explicit example (Bohm 1952), and Bell regarded it as such (Bell 1987). In the Schrödinger picture the state of two particles is a function on a six-dimensional configuration space, not a pair of functions on three-dimensional space, and a view in which that higher-dimensional space is the physical arena has been defended in the philosophy of physics (Albert 1996).

5.3 A real model: one state, two projections

To make the structure explicit we give a finite model using only real vectors and matrices. The state of the pair is a single vector of $\mathbb{R}^{12}$,

$$s = (x_1, y_1, z_1, t_1, x_2, y_2, z_2, t_2, o, e_1, e_2, r), \quad (5.2)$$

where $(x_1, \dots, t_1)$ and $(x_2, \dots, t_2)$ are the places and times at which the state is observed as particle A and particle B , $o \in \{0, 1\}$ records observation, e_1, e_2 are the displayed outcomes and $r = \pm 1$ is a random component fixed at the source. The two observers see the coordinate projections P_A , onto $(x_1, y_1, z_1, t_1, e_1)$, and P_B , onto $(x_2, y_2, z_2, t_2, e_2)$; each discards seven of the twelve components. The dynamics consists of three real matrices:

- separation, $S_u = I - uE_{x_1, t_1} + uE_{x_2, t_2}$, which moves only x_1 and x_2 and leaves o, e_1, e_2 and r unchanged;
- observation, which sets $o = 1$;
- coupling, $C = I + oN$ with $N = E_{e_1, r} - E_{e_2, r}$, $N^2 = 0$, so that $C = e^{oN}$.

Table 3 follows one state with $r = +1$, separated at $u = 1000$ m/s for 5 s.

Step	x_1	t_1	x_2	t_2	o	e_1	e_2	r
prepared	0	5	0	5	0	0	0	1
separated	-5000	5	5000	5	0	0	0	1
observed	-5000	5	5000	5	1	0	0	1
coupled	-5000	5	5000	5	1	1	-1	1

Table 3: One state of $\mathbb{R}^{12}$ through separation, observation and coupling ($y_i = z_i = 0$ throughout).

After coupling, $P_A s = (-5000, 0, 0, 5, 1)$ and $P_B s = (5000, 0, 0, 5, -1)$: ten kilometres apart, opposite outcomes, fixed by one matrix in one step. For $r = -1$ the outcomes are reversed. Applied before observation ($o = 0$), the coupling is the identity.

With measurement angles, one further component $\mu \in [0, 1]$ suffices: let $e_1 = r$, and $e_2 = -r$ if $\mu < \cos^2 \frac{a-b}{2}$, $e_2 = r$ otherwise. Averaging over r and μ gives $E(a, b) = -\cos(a - b)$ and hence $|S| = 2\sqrt{2}$. The model evades (5.1) because e_2 depends on a . The marginal law of e_2 is independent of a , so no signal passes from A to B .

5.4 Costs and status

The model is not offered as a new physical theory, and it makes no prediction that differs from quantum mechanics. Its purpose is to exhibit, in the simplest terms, the structure that Bell's theorem permits: correlations produced by a single state acting in one step in its own space, and appearing as action at a distance only in the two projections. Its costs are those of every such account. In three-dimensional terms it is non-local, and the order in which the coupling acts presupposes a notion of simultaneity for the two observations; a relativistic formulation would have to supply it or show that it is unobservable, as the de Broglie–Bohm theory does at the level of predictions. We do not claim to have met these costs; we claim only that the language of projection makes clear where they lie.

6. UNIFICATION AND THE SPACE OF COMPONENTS

The unifications of physics can be read as successive enlargements of the space of components.

Step	Unified	Enlargement
Maxwell 1865	electricity, magnetism, optics	a field with six components
Minkowski 1909	space and time; E and B	a fourth dimension; one antisymmetric tensor
Dirac 1928	relativity and the quantum electron	four spinor components
Kaluza 1921; Klein 1926	gravitation and electromagnetism	a fifth dimension
Glashow 1961; Weinberg 1967	electromagnetic and weak forces	internal components of each field
Witten 1995	the five string theories	an eleventh dimension

Table 4: Unification as enlargement.

The unification of gravitation with quantum theory has not been achieved. The pattern of Table 4 suggests, without proving, that the obstruction may lie in the space of components rather than in the dynamics: that we may be describing in four dimensions a state that does not fit there, as the roots of $x^2 + 4 = 0$ do not fit on the real line. For this suggestion to become physics it must be made testable. At a minimum, a proposal of this kind should specify the larger space, the projection onto observed quantities, and at least one consequence that differs from the four-dimensional description, as the Kaluza–Klein tower (4.1) does. Existing bounds (Section 4.2) already exclude a large class of such proposals, which is an argument for the programme's testability rather than against it.

7. CONCLUSION

Across kinematics, relativity, the electron, extra dimensions and entanglement, the same structure recurs: the state has more components than the observation, the observation is a projection, and anomalies in the projection point to the components it has discarded. Velocity is invisible in an image and is a column of a five-dimensional matrix; the electron's first-order equation requires

components that became spin and antimatter; a fifth dimension turns charge into momentum and one field into many masses; and an entangled pair, described as two objects, violates a bound that a single state observed twice need not obey. Physics has repeatedly progressed by enlarging the space in which it describes the world. There is no reason to think that the four dimensions in which we observe are the last such space.

REFERENCES

- Albert, D. Z. (1996). Elementary quantum metaphysics. In J. T. Cushing, A. Fine and S. Goldstein (eds), *Bohmian Mechanics and Quantum Theory: An Appraisal*, 277–284. Dordrecht: Kluwer.
- Anderson, C. D. (1933). The positive electron. *Physical Review* 43, 491–494.
- Appelquist, T., Cheng, H.-C. and Dobrescu, B. A. (2001). Bounds on universal extra dimensions. *Physical Review D* 64, 035002.
- Aspect, A., Dalibard, J. and Roger, G. (1982). Experimental test of Bell’s inequalities using time-varying analyzers. *Physical Review Letters* 49, 1804–1807.
- ATLAS Collaboration (2015). Summary of the searches for squarks and gluinos using $\sqrt{s} = 8$ TeV pp collisions with the ATLAS experiment at the LHC. *Journal of High Energy Physics* 10, 054. arXiv:1507.05525.
- Bell, J. S. (1964). On the Einstein Podolsky Rosen paradox. *Physics* 1, 195–200.
- Bell, J. S. (1987). *Speakable and Unspeakable in Quantum Mechanics*. Cambridge: Cambridge University Press.
- Bohm, D. (1952). A suggested interpretation of the quantum theory in terms of “hidden” variables, I and II. *Physical Review* 85, 166–193.
- Bohr, N. (1935). Can quantum-mechanical description of physical reality be considered complete? *Physical Review* 48, 696–702.
- Born, M. (ed.) (1971). *The Born–Einstein Letters*. London: Macmillan.
- Brauer, R. and Weyl, H. (1935). Spinors in n dimensions. *American Journal of Mathematics* 57, 425–449.
- Clauser, J. F., Horne, M. A., Shimony, A. and Holt, R. A. (1969). Proposed experiment to test local hidden-variable theories. *Physical Review Letters* 23, 880–884.
- Denavit, J. and Hartenberg, R. S. (1955). A kinematic notation for lower-pair mechanisms based on matrices. *Journal of Applied Mechanics* 22, 215–221.
- Dirac, P. A. M. (1928). The quantum theory of the electron. *Proceedings of the Royal Society A* 117, 610–624.
- Einstein, A. (1905). Zur Elektrodynamik bewegter Körper. *Annalen der Physik* 17, 891–921.
- Einstein, A., Podolsky, B. and Rosen, N. (1935). Can quantum-mechanical description of physical reality be considered complete? *Physical Review* 47, 777–780.
- Gibbs, J. W. (1902). *Elementary Principles in Statistical Mechanics*. New York: Scribner.
- Giustina, M. et al. (2015). Significant-loophole-free test of Bell’s theorem with entangled photons. *Physical Review Letters* 115, 250401.
- Glashow, S. L. (1961). Partial-symmetries of weak interactions. *Nuclear Physics* 22, 579–588.
- Hensen, B. et al. (2015). Loophole-free Bell inequality violation using electron spins separated by 1.3 kilometres. *Nature* 526, 682–686.
- İnönü, E. and Wigner, E. P. (1953). On the contraction of groups and their representations. *Proceedings of the National Academy of Sciences* 39, 510–524.
- Kaluza, T. (1921). Zum Unitätsproblem der Physik. *Sitzungsberichte der Preussischen Akademie der Wissenschaften*, 966–972.

- Klein, O. (1926). Quantentheorie und fünfdimensionale Relativitätstheorie. *Zeitschrift für Physik* 37, 895–906.
- Lee, J. G., Adelberger, E. G., Cook, T. S., Fleischer, S. M. and Heckel, B. R. (2020). New test of the gravitational $1/r^2$ law at separations down to $52\text{ }\mu\text{m}$. *Physical Review Letters* 124, 101101.
- Maxwell, J. C. (1865). A dynamical theory of the electromagnetic field. *Philosophical Transactions of the Royal Society* 155, 459–512.
- Minkowski, H. (1909). Raum und Zeit. *Physikalische Zeitschrift* 10, 104–111.
- Schrödinger, E. (1935). Discussion of probability relations between separated systems. *Mathematical Proceedings of the Cambridge Philosophical Society* 31, 555–563.
- Shalm, L. K. et al. (2015). Strong loophole-free test of local realism. *Physical Review Letters* 115, 250402.
- Tsirelson, B. S. (1980). Quantum generalizations of Bell’s inequality. *Letters in Mathematical Physics* 4, 93–100.
- Weinberg, S. (1967). A model of leptons. *Physical Review Letters* 19, 1264–1266.
- Witten, E. (1995). String theory dynamics in various dimensions. *Nuclear Physics B* 443, 85–126.